Computer Simulations to Accompany the paper

Social Relations Analysis of Dyadic Data Structures:

The General Case

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Simulations were conducted to assess the validity of the Bond and Malloy ARBSRM algorithm for estimating the Social Relations Model from non-standard dyadic structures. Of interest were the parameter estimates generated by ARBSRM, as well as ARBSRM estimates of the standard errors for those parameter estimates. With these statistics, we developed significance tests for SRM parameters, and our simulations assessed those too.

Method

Five dyadic structures were investigated: a complete 8 x 8 round-robin, an incomplete 10 x 10 round robin with concentrated symmetric missing dyads, an incomplete 10 x 10 round-robin with diffuse symmetric missing dyads, an asymmetric incomplete 10 x 10 round-robin, and a very asymmetric incomplete 10 x 10 round-robin. For the five dyadic structures, see Table 1. Each dyadic structure is represented by a matrix. Actors are represented by the rows in each matrix. Partners are represented by the columns. Consistent with Social Relations convention, self-scores are missing from all of our dyadic structures. Each self-score is represented by an x. An entry of 1 means that a data point is present in a given dyadic structure. A entry of 0 means that the data point is missing.

Let us discuss the five Dyadic Structures. In a round robin, each individual in a group interacts one-on-one with every other individual in the group. Two data points are extracted from the interaction between Person i and Person j. In one, Person i functions as an Actor and Person j as a Partner. In the other, Person j functions as an Actor and Person i functions as a Partner. Round robins have been used in the bulk of all Social Relations Model research to date. Round robin data can be analyzed with existing methods. David Kenny's program SOREMO provides estimates of SRM population parameters when applied to round robin data. See Kenny, Kashy, and Cook (2006). Bond and Lashley (1996) devised exact and estimated theoretical standard deviations for SRM parameter estimates from round robin data. These can be computed with existing software: the TripleR computer program (Schonbrodt, Back, & Schmukle, 2012). Significance tests and statistical power computations for SRM analyses of round robin data are available (Lashley & Bond, 1997, Lashley & Kenny, 1998). Although ARBSRM is not needed for Social Relations analysis of round robin data, we included it in our simulation as a baseline comparison standard for analyses of other dyadic structures. Note that there are 56 data points on each variable in bivariate 8 x 8 round robin data.

Our goal was to assess the validity of the ARBSRM algorithm in its application to non-standard dyadic structures. We chose to simulate four non-standard bivariate dyadic structures – two symmetric structures and two non-symmetric structures. Each of our non-standard dyadic structures includes 56 data points on each of two variables – just like the 8 x 8 round robin. Our symmetric concentrated structure involves ten individuals. It includes a complete round robin among five of those individuals, and no interactions at all among five individuals. Thus relative to a round robin, there is missing data concentrated among a subset of 10 individuals. Our diffuse symmetric structure also includes ten individuals. Here there are data points symmetrically missing in a diffuse manner across the structure. One individual interacts with only two other individuals; and each of two other individuals interact with eight others. We also simulated two structures that are not symmetric. One we will call asymmetric and the other very asymmetric. Note that in both of these latter structures individual 10 contributes data points as a Partner while contributing no data points as an Actor. In practice, this might occur in a two-session study, by virtue of an individual who was present at one of the sessions but not the other. Note that in the asymmetric structure, 28 of the 56 data points are not mirrored – e.g. X[i,j] is present but X[j,i] is missing. Thus, our so-called "asymmetric" dyadic structure is half asymmetric. In the very asymmetric structure, 34 of the 56 data points present are not mirrored – X[i,j] being present when X[j,i] is missing. It may also be instructive to regard the very asymmetric structure as a 10 x 10 round robin with missing data. There would be 90 data points in a 10 x 10 round robin. In our very asymmetric structure, <u>every</u> data point missing from the round robin is asymmetrically missing.

Bivariate data were generated from populations that had known values. The relevant population parameters are the sixteen in the bivariate Social Relations Model – five variance/covariance parameters for one variable (X), five for a second variable (Y), and six covariance parameters for the relationship between X and Y.

We simulated effects from nine different configurations of SRM population parameter values. Two of the configurations included non-zero values for individual-level parameters only, two included non-zero values for dyadic-level parameters only, and five included non-zero values for both individual-level and dyadic-level parameters. For the nine parameter configurations, see Table 2. There each parameter configuration is represented by a column of sixteen values. We will be calling the first column of values Configuration 1, the second Configuration 2, …, and the ninth column of parameter values Configuration 9.

For purposes of these simulations, we chose values for SRM parameters in the following way. Non-zero values for the parameters of variable X were chosen to correspond to values used in the simulations by Lashley and Bond (1997). Non-zero values for the parameters of variable Y were chosen to be different than those for X. We set covariances to values that would produce correlation coefficients of +.5 and -.5, with a few exceptions. For generality, both positive and negative covariances were included. Parameter Configurations 6-8 represent a crossing of the non-zero values in the individual-level parameters of Configurations 1 and 2 with the non-zero values in the dyadic-level parameters of Configurations 3 and 4. In our simulation plan for Configurations 1-8, we set many SRM variances and covariances to 0. To complement that plan, we set every SRM variance and covariance to a non-zero value for parameter Configuration 9.

We conducted these simulations with the R programming language. Each parameter Configuration was used to generate observations conforming to each dyadic Structure in each of 20,000 data sets. Effects were drawn from multivariate normal distributions with two variance/covariance matrices specified by the Configuration. One was a 4 x 4 variance-covariance matrix for the Actor effect on X, the Partner effect on X, the Actor effect on Y, and the Partner effect on Y. The other was a variance-covariance matrix for four relationship effects: the [i,j] relationship effect on X, the [j,i] relationship effect on X, the [i,j] relationship effect on Y, and the [j,i] relationship effect on Y. We generated multivariate normal effects with the R function mvrnorm. Effects were summed to form observed scores. In each data set, there were 56 observed scores on each of the two variables. Once 20,000 data sets had been generated and analyzed for a given parameter Configuration and dyadic Structure, the mean estimate for each parameter in the 20,000 data sets was noted, as was the observed standard deviation (across the 20,000 data sets) in each estimate. Also noted was the square root of the mean of the Bond and Malloy theoretically estimated variance in each parameter estimate.

From the analysis of each simulated data set, we formed the ratio of each variance/covariance estimate to its theoretical estimated standard error. 900,000 values of this statistic were noted for each of the 16 bivariate SRM parameters – one from each of the 20,000 data sets in each of 45 simulation cases (5 Dyadic Structures x 9 Parameter Configurations). This statistic was used to test the hypothesis that a given SRM parameter equaled zero. I referred each value of this test statistic to two different criteria. One set of analyses noted whether a given estimate was statistically significant, relative to a critical t with 7 degrees of freedom (= 8 individuals minus 1). Another noted whether the estimate was statistically significant, relative to a critical t with 55 degrees of freedom (= 56 observations – 1). The alpha level for every significance test was .05. Following Social Relations conventions, tests of variances were one-tailed because variances cannot be negative. Tests of covariances were two-tailed. The $t(7)$ significance tests are the tests devised by Lashley and Bond (1997) for round robin data. Lashley and Bond's simulations indicated that those tests were sometimes low in statistical power. Thus we investigated a test that might offer more power, one that referred our test statistic to a more liberal criterion: $t(55)$.

All of these results were recorded for each of the 16 SRM parameters from an initial batch of 20,000 simulated data sets for each case in our 5 (Dyadic Structures) x 9 (Parameter Configuration) simulation plan.

Later, we ran a second set of simulations, with the same five Dyadic Structures and same nine Parameter Configurations. A new batch of 20,000 data sets for each of these 5 x 9 cases was generated. Again, we noted from each of those 20,000 data sets the mean estimate of each SRM parameter, the observed standard deviation in estimates of the parameter, the square root of the mean of our theoretically derived variance in the parameter estimate, results of the $t(7)$ significance tests and the $t(55)$ significance tests. We call our first group of data sets Batch A and our second group of data sets Batch B.

Results

Here we present the results of our simulations. First, we discuss our SRM parameter estimates. With simulation results, we demonstrate that the ARBSRM algorithm produces unbiased estimates of SRM variances and covariances. Next, we discuss our theoretically derived estimates of standard errors in these SRM parameter estimates. As the simulation results show, our theoretical estimates of standard errors in SRM parameter estimates are extremely close to the standard deviations we observe in those estimates over large numbers of data sets. Finally, we discuss two significance testing procedures for SRM parameters. As results show, a liberal testing procedure is invalid in producing too many Type I errors. A more conservative procedure is valid in the vast majority of cases, though invalid in testing for Actor-Partner Covariance in very asymmetric data.

Mean Parameter Estimates

The procedures we used are intended to provide unbiased estimation of SRM variances and covariances. If our estimates are, in fact, unbiased, then the mean of a large number of SRM parameter estimates should be close to the known value of the population parameter it is estimating.

We checked for unbiasedness in our estimators in several ways. For each of the sixteen bivariate SRM parameters, we had 900,000 estimates – 40,000 from each of 45 cases (5 Dyadic Structures x 9 Population Parameter Configurations. We noted the deviation within each case between the mean of a parameter estimate over 40,000 data sets and the corresponding population value. Mean values of these deviations over the 9 Configurations appear in the left column of Table 3, for each bivariate SRM parameter separately. As we see there, the means of our SRM parameter estimates are quite close to population values – largest mean deviation is for Dyadic Covariance in Y, where the mean estimate is .0072 lower than the estimate.

Although these values appear small, they may be hard to interpret. More interpretable, we suspect, would be a measure of the deviation of mean estimates away from population parameters, as a percentage of the value of the population parameter. For each relevant case, we computed the mean deviation of 40,000 SRM parameter estimates from the parameter being estimated, as a percentage of the value of that parameter – confining attention to cases where the parameter value was not zero. Relevant results appear in the second column of Table 3. Most mean deviations are less than 1% of the quantity being estimated. The largest is for the covariance between the Actor effect in X and the Partner effect in Y. The mean estimate there is 2.59% higher than the parameter value.

We were also interested in how close these sample means were from population values, irrespective of arithmetic sign. Thus we noted (in each group of 40,000 data sets) the absolute value of the mean deviation from its population value, and aggregated these mean absolute deviations over the 45 simulation cases. Again, we confined attention to cases where the parameter being estimated was not zero and expressed the absolute mean deviation as a percentage of the value of the parameter. Results appear in the third column of Table 3. Most mean absolute deviations are less than 1% of the quantity being estimated. The largest absolute deviation is again for the covariance between the Actor effect in X and the Partner effect in Y. The mean estimate there for the one relevant parameter configuration is 2.59%, as above.

We wondered if our ARBSRM parameter estimates would be unbiased if applied to data from each of the five Dyadic Structures we investigated. Confining attention to cases where the population parameter being estimated was non-zero, mean ARBSRM estimates deviated from population parameters by +0.01%, -0.04%, +0.03%, +0.01%, and -0.16% when the estimates were made from round robin, concentrated symmetric, diffuse symmetric, asymmetric, and very asymmetric data, respectively. In absolute terms, means deviated from population parameter values by 0.33%, 0.37%, 0.34%, 0.48%, and 0.63% of the values of those parameters when computed from those Dyadic Structures. In general, these deviations are small, but they are larger when SRM parameters are estimated from asymmetric and very asymmetric data.

The data we have been discussing aggregate mean estimation errors across all sixteen bivariate SRM parameters. To understand the data, it is important to consider a distinction among bivariate SRM parameters. Most SRM parameters concern a within-effect relationship – an actor effect in its relationship to an actor effect, a partner effect in its relationship to a partner effect, or a dyadic [i,j] effect in its relationship to a dyadic [i,j] effect. Other SRM parameters concern a cross-effect relationship – an actor effect in its relationship to a partner effect, or a dyadic [i,j] effect in its relationship to a dyadic [j,i] effect. It occurred to us that cross-effect parameter estimates depend heavily on mirrored data points, and that are fewer such data points in non-symmetrical than symmetrical Dyadic Structures.

Pursuing this logic, we examined mean absolute estimation error from our three symmetric Structures with the corresponding error in our two non-symmetric Structures for three types of parameters: within-effect parameters, actor-partner covariances, and dyadic covariances. Results appear in Table 4. As we see there, mean absolute percentage estimation errors for within-effect parameters are similar whether the estimates are made from symmetric or non-symmetric data; for the difference, $t(57) = .17$, *n.s.* Mean absolute errors are greater for estimates made from non-symmetric data than symmetric data, when the quantity being estimated is a cross-effect parameter; for the non-symmetric vs. symmetric difference, $t(57) = 5.79$, $p < .001$. This, we believe, reflects the relative dearth of mirrored data points in non-symmetric Dyadic Structures.

These are deviations of mean sample estimates from population values. Many of these estimation errors strike us as small, and a few seem less small. Although it may be useful to know the absolute (and relative) size of the estimation errors the ARBSRM algorithm yields, our primary goal is to validate these estimation procedures. We hope to show that ARBSRM produces <u>unbiased</u> estimators of SRM variances and covariances. To do so, we need to remind the reader that, we have simulated 40,000 data sets for each of 45 cases (9 Parameter Configurations x 5 Dyadic Structures). We have recorded the mean estimate of each of 16 SRM parameters across the 40,000 data sets for each case. We have also observed (and recorded) the standard deviation across the set of 40,000 estimates for each case.

If our estimators of an SRM population parameter are unbiased, the mean of 40,000 sample estimates of an SRM parameter should not deviate from the population value of that parameter value any more than would be produced by chance. To test this logic, we created 144 confidence intervals for data from each of five dyadic structures. The 144 confidence intervals included estimates of 16 different SRM parameters from each of nine different Parameter Configurations. Each confidence interval was created around the mean of 40,000 estimates of an SRM parameter. We created a 95% confidence interval around that mean, in the following way.

Lower limit of CI $= M - 1.960 * s_M$

Upper limit of CI $= M + 1.960 * s_M$

where M represents the mean of 40,000 ARBSRM parameter estimates,

1.960 is the value in a t(39999) distribution below which 97.5% of observations fall, and

s_M is the standard error of the mean – that is the observed standard deviation of the

40,000 parameter estimates divided by the square root of 40,000.

If our estimators are unbiased, no more than 5% of confidence intervals constructed in this way should fail to include the value of the population parameter. Results show that of 144 confidence intervals, 3.47% of those constructed from round robin data, 4.17% constructed from concentrated symmetric data, 5.56% constructed from diffuse symmetric data, 4.17% constructed from asymmetric data, and 5.56% constructed from very asymmetric data fail to include the known population value being estimated. As these results indicate, sampling variability can account for the deviations of our mean parameter estimates from population parameter values. Although two of these 95% percent confidence interval fail slightly more than 5% of the time, the deviations above 5% parallel those below 5% and are within a standard deviation of expectation, a binomial analysis indicates. In the absence of evidence for extra-chance deviations of mean estimates from population values, we conclude that these estimators are unbiased.

Observed and Theoretical Variability in Parameter Estimates

Based on work in Searle, Casella, and McCulloch (1992), Bond and Malloy devised unbiased estimates of the variance in SRM parameter estimates. These are based on the assumption that actor, partner, and relationship effects are drawn from normal population distributions, as they are in the simulations we are reporting. From two independent batches of simulated data, various statistical results were noted. Of current interest are the observed standard deviation in ARBSRM estimates of a population variance/covariance parameter across 20,000 data sets, and the square root of the mean theoretically derived estimate of the variance in those parameter estimates. The latter is the mean of 20,000 values.

Note that the square root of the mean of 20,000 unbiased theoretical estimates of the variance of an SRM parameter estimate should be very close to the exact standard deviation in those parameter estimates. If Bond and Malloy's theoretical standard deviation estimates are valid, the following should be true:

a) Theoretical estimates of the standard deviation in SRM parameter estimates should be close to the observed standard deviation in those estimates across 20,000 data sets, and

b) these theoretical estimates of the standard deviation in SRM parameter estimates should be strongly correlated with observed standard deviations in those estimates when there is systematic variation in each.

An obstacle to assessing the validity of these standard errors is sampling variability. To get a preliminary notion of the magnitude of sampling variability in the variables of interest, we divided 40,000 values of each variable for each bivariate SRM parameter in half, and noted the absolute difference between the values of each variable in one batch of 20,000 data sets (Batch A), as opposed to another batch of 20,000 data sets (Batch B).

The mean results for the absolute difference between a value in Batch A and the corresponding values in Batch B appear in the first two columns of Table 5. As we see there, there are small differences between standard deviations in parameter estimates observed in one large batch of data and the corresponding standard deviations observed in a second large batch (mean absolute differences on the order of .01-.03). The difference between theoretically based estimates of the standard deviation in parameter estimates in one large data batch vs. the other is even smaller (mean absolute differences on the order of .005-.01). Having some idea of the stability of observed and theoretical standard deviations in ARBSRM parameter estimates across two large Batches of data, let us compare standard deviations that are empirically observed in 20,000 data sets with theoretically estimated standard deviations. It will be of interest to make this comparison across data Batches. The mean absolute difference between the standard deviation observed in a case of data Batch A and the standard deviation theoretically estimated for the corresponding case of Data Batch B appear in the third column of Table 5, and the mean absolute difference between the theoretically estimated standard deviation for a case in data Batch A and the standard deviation empirically computed for the corresponding case in Batch B appear in the fourth column.

It is noteworthy that the values in the last two columns of Table 5 are smaller than the values in the first column. To assess the statistical significance of this difference, we noted the mean of the five quantities in column 1 and the mean of the ten quantities in columns 3 and 4. The former is significantly greater than the latter, by a paired-comparison t-test across the 144 simuation cases (16 SRM parameters x 9 Parameter Configurations). For the difference, $t(143) = 106.89$, $p < .0000001$.

As these results show, the standard deviation theoretically estimated from data in one Batch is closer to the standard deviation empirically observed in data from another Batch, than are the two empirically observed standard deviations to one another. Our theoretical estimates of standard deviations err less from empirically observed standard deviations than empirically observed standard deviations err from one another.

Although Table 5 provides data on the difference between theoretically estimated and empirically observed standard errors, the differences may be hard to interpret. To enhance the interpretability of these differences, we began by aggregating theoretical and observed standard deviations across the two data Batches. We then noted the mean absolute difference between a theoretically estimated standard deviation and the same empirically observed standard deviation, and expressed this difference as a percentage of the value of the empirically observed standard deviation. We did this separately for data from each of our dyadic structures. Results show that (as a percentage of the empirically observed standard deviation) theoretical estimates of standard deviations differ from their empirical counterparts by 0.20%, 0.30%, 0.25%, 0.28%, and 0.25% when computed from round robin, concentrated symmetric, diffuse symmetric, asymmetric, and very asymmetric data. All mean absolute differences are below one third of 1%.

One wonders if the variability of SRM parameter estimates depends on the Dyadic Structure of the data. They are. See Table 6 for theoretical and observed standard deviations in estimates from each of five Dyadic Structures. Note that SRM parameter estimates from round robin, concentrated symmetric, and diffuse symmetric data have lower variability than estimates from asymmetric and very asymmetric data. Estimates from asymmetric and very asymmetric data are especially variable when the parameter being estimated is an Actor-Partner Covariance or a Dyadic Covariance. For relevant results, see the parenthesized standard deviations back in Table 4. Analyses on those data show that standard deviations in parameter estimates are higher in non-symmetric than in symmetric Dyadic Structures – whether the parameter being estimated is within-effect or between-effect; for the symmetric vs. non-symmetric difference, t (143) = 16.62 and 18.43 for within-effect and between-effect parameters, respectively; each $p < .0001$. However, an ANOVA showed that the symmetric vs. non-symmetric difference in standard deviations was greater for between-effect than within-effect parameters; for the interaction, F (1,142) = 177.79, $p < .0001$.

Overall, we have 144 theoretically estimated standard errors and 144 observed standard deviations from round robin data (one for each of the 16 SRM parameters in each of the 9 population configurations). Similarly we have 144 theoretically estimated standard errors and 144 observed standard deviations from concentrated symmetric data, from diffuse symmetric data, from asymmetric data, and from very asymmetric data. For purposes of comparing our theoretical standard deviations with observed standard deviations, we began by aggregating these variables across data Batch A and data Batch B. We then correlated our theoretically estimated standard deviation with the observed standard deviation across our 144 (16 x 9) cases, for simulations of round robin, concentrated symmetric, diffuse symmetric, asymmetric, and very asymmetric data. Results show that within each of the five dyadic structures, our theoretical standard deviations are almost perfectly correlated with observed standard deviations across the 144 cases. The lowest of five correlations yielded an $r > .9999$.

It might be useful in evaluating these correlations, to restrict attention to cases where the population parameter value in question was 0. Analyses show that both theoretical and observed standard deviations are lower when the parameter being estimated is zero, rather than non-zero.

Of our 144 simulation cases (of 16 SRM parameters in each of 9 population parameter configurations), 86 were ones where the population parameter being estimated was equal to zero. We noted the correlations between our theoretically derived standard errors and observed standard deviations across these 86 cases. Again, simulations from each of the five data structures invariably showed that our theoretically derived standard errors were almost perfectly correlated with observed standard deviations (again, lowest $r > .9999$).

The correlations above incorporate estimates of sixteen different SRM parameters. Perhaps it is not sensible to correlate standard deviations across parameters. Perhaps it would be more meaningful to assess theoretical and observed standard deviations separately for each of the SRM parameters – correlating them once in estimates of Actor Variance in X, once in estimates of Partner Variance in X, …, and once in estimates of Dyadic Covariance across X and Y. We computed the relevant 90 correlation coefficients – round robin theoretical with round robin observed standard deviations, concentrated symmetric theoretical with concentrated symmetric observed standard deviations, diffuse symmetric theoretical with diffuse symmetric observed standard deviations, asymmetric theoretical with asymmetric observed standard deviations, and very asymmetric theoretical with very asymmetric observed standard deviations within estimates of each of the 16 SRM parameters separately. The lowest of these 90 correlations yielded an $r > .9999$.

Across two large Batches of data, our theoretical estimates of standard deviations in SRM parameter estimates are (on the average) closer to empirically observed standard deviations than are empirically observed standard deviations to themselves. Across all of our simulation cases and all subsets of those cases we examined, our theoretical standard deviations correlate near perfectly with empirically observed estimates. As these results indicate, the ARBSRM algorithm provides unbiased estimates of the standard deviation in SRM variance/covariance estimates.

ARBSRM Hypothesis Tests

We set up a statistic to test the null hypothesis that an SRM population parameter equals 0. This statistic is our ARBSRM estimate of that parameter divided by the square root of the ARBSRM theoretical variance in that parameter estimate. For each of our 144 cases (16 SRM parameters x 9 population parameter configurations) and each of our five dyadic structures, this ratio was computed 40,000 times – from 40,000 different parameter estimates and 40,000 different values of the theoretical standard deviation in a parameter estimate. For purposes of hypothesis testing, we compared this test statistic to two criteria: a critical *t* with 55 degrees of freedom, and critical *t* with 7 degrees of freedom. The nominal alpha level was .05 in every hypothesis test. We focus on the Type I error rate of our hypothesis tests. Of the 144 cases in our simulations (16 x 9), eighty six were cases in which the parameter of interest had a population value of zero. In each of those cases, we tested the hypothesis that this parameter equaled zero 200,000 times – 40,000 hypothesis tests for each of our five Dyadic Structures. If our testing procedure is valid, our tests should reject null hypotheses no more than 5% of the time. We tabulated empirical null hypothesis rejection rates separately for testing of data from our five Dyadic Structures. Let us consider first hypothesis tests that compare our test statistic against a critical value in a *t* distribution with 55 degrees of freedom. Results showed that in many cases, true null hypotheses were rejected less than 5% of the time. Yet in tests from round robin data, seven of eighty-six cases yielded more than 6% Type I errors. Concentrated symmetric, diffuse symmetric, asymmetric, and very asymmetric data yielded more than 6% Type I errors in eight, ten, eight, and thirty-six of the eighty-six cases. A number of these empirical Type I error rates were well above 6%. In eighteen of these cases, tests that had a nominal Type I error rate of .05 rejected true null hypotheses more than 9% of the time. These tests cannot be recommended. They produce too many Type I errors. Very asymmetric data is especially vulnerable to inflated Type I error rates, given a t(55) critical cut-off value.

We also simulated a more conservative significance test. Again, we form the ratio of an ARBSRM parameter estimate to its theoretically estimated standard deviation. We then compare this statistic to a critical value in a t distribution with 7 degrees of freedom. This is the testing procedure investigated by Lashley and Bond (1997) for round robin data.

Looking again at empirical Type I error percentages in 40,000 hypothesis tests for each Dyadic Structure and each of 86 null cases, the following results emerged. Tests on round robin data yielded less than 5% Type I errors in 84 of 86 cases. In one of the nine parameter configurations (configuration 4), there were 6.09% Type I errors in tests of the covariance between Actor effects in one variable and Actor effects in the other variable, and 6.17% Type I errors in tests of the covariance between Partner effects in one variable and Partner effects in the other variable. Tests on concentrated symmetric data yielded less than 5% Type I errors in 84 of 86 cases. Concentrated symmetric tests of the Relationship variance in Y showed excessive Type I errors in population parameter configurations 1 and 2. There, 10.30% and 10.67% of true null hypotheses were rejected. Tests on diffuse symmetric data performed similarly. They yielded less than 5% Type I errors in 84 of 86 cases. Diffuse symmetric tests of the Relationship variance in Y yielded 6.97% and 7.13% Type I errors, in parameter configurations 1 and 2 (as with concentrated symmetric tests). Tests on asymmetric data yielded less than 5% Type I errors in 84 of 86 cases. Asymmetric tests of the Relationship variance in X yielded 7.71% and 8.10% Type I errors in parameter configurations 1 and 2, respectively. Tests on very asymmetric data did not perform well. These tests yielded less than 5% Type I errors in only 70 of 86 cases. In eleven cases, the tests rejected true null hypotheses more than 6% of the time. In four different parameter configurations, very asymmetric tests of the Covariance between the Actor and Partner effect in X rejected 6.44%, 6.50%, 8.25%, and 8.46% of true null hypotheses. In four parameter configurations, very asymmetric tests of the Covariance between Actor and Partner effect in Y rejected 6.95%, 7.19%, 7.33%, and 8.61% true null hypotheses. There were also excessive Type I errors in tests of the bivariate Covariances between the Actor effect in one variable and the Partner effect in the other variable.

Overall, the more conservative testing procedure appears to exert adequate control over Type I error rates in most analyses of round robin, concentrated symmetric, diffuse symmetric, and asymmetric data. Occasionally, our simulations suggested excessive Type I errors in tests of those dyadic structures. It is noteworthy that these excess Type I error rates were in tests of the statistical significance of Relationship variance. In certain contexts, Social Relations researchers have little interest in Relationship variance because they cannot separate it from random error. In other contexts, Relationship variance is of interest. In any event, ARBSRM hypothesis tests of Relationship variance should be viewed with caution. This more conservative testing procedure does not adequately control Type I errors when applied to very asymmetric data. In certain tests of very asymmetric data, excessive Type I errors can be expected. These are tests of one of the cross-effect SRM parameters we isolated in Table 4. In analyses of dyadic structures that include only a few mirrored data points, claims of statistical significance in actor-partner covariance cannot be trusted.

An Alternative: TripleR

Having assessed the Bond and Malloy ARBSRM algorithm for Social Relations analysis of nonstandard data, we hoped to assess an alternative. Schonbrodt, Back, and Schmukle (2012) developed procedures for the Social Relations analysis of round robin data. They implemented these procedures in an R package called TripleR. For the Social Relations analyst, this TripleR package is attractive in many ways. It can handle both univariate and bivariate Social Relations analyses, analyses on manifest and latent variables, analyses of data from a single group and from multiple groups. TripleR outputs both SRM parameter estimates and standard errors for those estimates (Bond & Lashley, 1996).

It can provide estimates of individuals' actor and partner effects and several other Social Relations results. See Schonbrodt, Back, and Schmukle (2012) for details.

Of special interest in the current context, TripleR can be used to analyze nonstandard dyadic data structures. Having been designed for round robin data, TripleR treats any departure from a round robin as missing data. It can impute these missing values, and analyze the round robin that is completed once the imputations are made.

We assessed TripleR's Social Relations analytic abilities with data from our five dyadic structures: the complete 8 x 8 round robin data, incomplete concentrated symmetric data, incomplete diffuse symmetric data, asymmetric data, and very asymmetric data. To do so, we had TripleR analyze some of the data ARBSRM had analyzed – data Batch B. We had stored that data Batch – 20,000 10 x 10 bivariate round robins randomly generated from each of our nine SRM population parameter Configurations. As before, we abstracted five subsets of the data from each 10 x 10 round robin – one corresponding to each of the five dyadic structures of interest. Earlier we reported ARBSRM analyses of these data. Now we report TripleR analyses.

Schonbrodt, Back, and Schmukle (2012) designed TripleR to analyze round robin data. Let us comment on how it does so. We had TripleR analyze a total of 180,000 bivariate 8 x 8 round robins in data Batch B. From each analysis, we noted TripleR's estimate for each of 16 SRM population parameters. We compared those parameter estimates to the estimates made earlier by ARBSRM. The two sets of estimates were identical. They were identical in 180,000 of 180,000 analyses. When applied to round robin data, ARBSRM and TripleR yield the same SRM variance/covariance estimates.

Each time ARBSRM estimates an SRM variance/covariance, it reports a standard error for that estimate – a standard error of the form devised by Bond and Lashley (1996). TripleR reports standard errors for some of its SRM parameter estimates but not others. TripleR does not, for example, report a standard error for an estimate of Actor variance, if the estimate is negative. Thus, it was not always possible for us to compare TripleR's standard errors with ARBSRM's standard errors. However, in every case where TripleR reported a standard error from our 8 x 8 data Batch B round robin, it was identical to ARBSRM's standard error.

Above, we proposed a statistic for testing the hypothesis that an SRM population parameter was equal to zero. This was the ARBSRM estimate of that parameter divided by its (ARBSRM-computed) standard error. When applied to round robin data, TripleR yields the same SRM parameter estimates as ARBSRM, and (whenever it reports a standard error), TripleR reports the same value for that standard error as ARBSRM. Thus in analyzing round robin data, TripleR and ARBSRM offer identical tests of statistical significance – whenever TripleR reports such a test.

For analyzing round robin data, we invite readers to use TripleR. For round robin data, David Kenny's program SOREMO is also a good choice. Our interest is elsewhere -- in the analysis of non-standard dyadic data. Those are the analyses for which we created ARBSRM. Although TripleR was not designed to analyze non-standard dyadic data, it has the ability to do so. It does so by treating any departure from a round robin structure as missing data. It imputes missing values and analyzes the completed round robin.

Our simulations feature dyadic structures that differ markedly from the round robin. Looking back at Table 1, we see two symmetric structures that could be viewed as incomplete 10 x 10 round robins. Each structure would become a 10 x 10 round robin if 34 data points were added. Thus, TripleR regards each of our incomplete symmetric structures as a 10 x 10 round robin with 34 (of 90) data points missing. Looking back over the two non-symmetric dyadic structures in Table 1, let us note that TripleR treats each of these as a 9 x 9 round robin, omitting all data involving individual 10 and imputing 24 (or 25) data points to complete the 72-observations in the nine-person round robin.

TripleR was not designed to analyze data from nonstandard dyadic structures like the ones we are simulating here. Indeed, Schonbrodt, Back, and Schmukle (2012) warn researchers against using

TripleR in this way. Even so, we hoped to assess an alternative method for the Social Relations analysis of nonstandard dyadic data, and wondered how TripleR would fare. Perhaps despite the warnings, TripleR will perform as well as ARBSRM in analyzing nonstandard dyadic structures. If so, we would endorse it over ARBSRM. Relative to ARBSRM, Triple R is more convenient, flexible, and widely-distributed.

Earlier, we noted that ARBSRM was unbiased in its estimation of SRM parameter estimates. It was unbiased in estimating those parameters from round robin data, and also from data that conformed to each of our nonstandard data structures. TripleR is unbiased in estimating SRM parameters if it is analyzing round robin data. Indeed, it gives the same estimates as ARBSRM. However, when analyzing nonstandard dyadic data, it is biased. We noted this bias by computing the mean of 20,000 TripleR estimates of each of 16 SRM parameter estimates across each of 9 population parameter Configurations, and comparing those mean estimates to the corresponding population values. As in our analyses of ARBSRM above, we confined attention to non-zero population values, and expressed bias as a percentage of those population values. In general, TripleR underestimates SRM parameters. Averaging over all 58 relevant cases, TripleR's bias in estimating non-zero population values ranged from -33.65% (when computed from asymmetric data) to -42.08% (when computed from very asymmetric data).

Although in general TripleR underestimates SRM population parameters in our simulations, there are three parameters that it overestimates. When analyzing our nonstandard data, Triple R overestimates Actor-Partner covariance in X, Dyadic covariance in X, and Relationship XY covariance. In the population, those parameters had values of -.707, -1.75, and -8.282. TripleR estimated those quantities (on the mean) to be -.415, -.880, and -3.920. These mean estimates are 41.22%, 49.73%, and 52.65% too high. In analyses of nonstandard data, TripleR does not consistently underestimate population parameters. Rather, it estimates the parameter to be closer to 0 than it is.

In describing their program TripleR, Schonbrodt, Back, and Schmukle (2012) discuss the issue of missing data. They advise users to expect relatively unbiased results if in a 10 x 10 round robin there are less than 11 data points missing. Our nonstandard dyadic structures feature more than 11 missing data points. In such cases, Schonbrodt, Back, and Schmukle (2012) lead us to expect that "with an increasing proportion of missing values, the relationship variance will be underestimated, and the actor and partner variances will be overestimated."

We assessed this expectation with TripleR's estimates of Actor variance, Partner variance, and Relationship variance from our nonstandard data. Results with non-zero population parameter values show that TripleR underestimated Relationship variances by 51.90%, underestimated Actor variances by 28.14%, and underestimated Partner variances by 22.68%. In our simulations, missing values lead Actor variance and Partner variance to be underestimated, contrary to Schonbrodt, Back, and Schmukle's claim.

From each TripleR analysis of 20,000 nonstandard data sets, we observed the standard deviation across estimates of each of the 16 bivariate SRM parameters. In conjunction with some of its parameter estimates (but not others), TripleR reported a standard error (Bond & Lashley, 1996). We wanted to assess the correspondence between the theoretical standard errors and the observed standard deviations.

In TripleR analyses of nonstandard data, the theoretical standard errors did not provide good estimates of empirically observed standard deviations. The theoretical standard errors that were reported underestimated the corresponding observed standard deviations in parameter estimates by 23.45% to 29.17% (when computed from asymmetric and very asymmetric data, respectively).

While reporting certain SRM parameter estimates (but not others), TripleR reports the value of a t-ratio and its statistical significance. We wished to assess the significance tests TripleR reported for cases where the null hypothesis about an SRM population parameter was true. In significance tests on true null hypotheses with concentrated symmetric data and diffuse symmetric data, TripleR had an empirical Type I error rate of over 25% 20 of 172 times. Asymmetric and very asymmetric Triple R tests each had an empirical Type I error rates over 25% 16 of 172 times. In its tests of Relationship variance in X and Relationship variance in Y, TripleR rejected true null hypotheses 100% of the time. It rejected every one of those true null hypotheses in every analysis of every nonstandard dyadic structure we simulated.

Overall, TripleR is a fine program for analyzing round robin data, but cannot be recommended for the analysis of nonstandard dyadic data structures. For the latter, we have developed ARBSRM.

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Table 1

Five Dyadic Structures

Round Robin

		Partner							
		1	2	3	4	5	6	7	8
	1	x	1	1	1	1	1	1	1
	2	1	x	1	1	1	1	1	1
	3	1	1	x	1	1	1	1	1
Actor	4	1	1	1	x	1	1	1	1
	5	1	1	1	1	x	1	1	1
	6	1	1	1	1	1	x	1	1
	7	1	1	1	1	1	1	x	1
	8	1	1	1	1	1	1	1	x

Concentrated Symmetric

		Partner									
		1	2	3	4	5	6	7	8	9	10
	1	x	1	1	1	1	1	1	1	1	1
	2	1	x	1	1	1	1	1	1	1	1
	3	1	1	x	1	1	1	1	1	1	1
	4	1	1	1	x	1	1	1	1	0	0
Actor	5	1	1	1	1	x	0	0	0	0	0
	6	1	1	1	1	0	x	0	0	0	0
	7	1	1	1	1	0	0	x	0	0	0
	8	1	1	1	1	0	0	0	x	0	0
	9	1	1	1	0	0	0	0	0	x	0
	10	1	1	1	0	0	0	0	0	0	x

Diffuse Symmetric

		Partner									
		1	2	3	4	5	6	7	8	9	10
	1	x	0	1	1	1	0	1	0	1	0
	2	0	x	0	1	0	1	1	1	1	0
	3	1	0	x	1	1	1	1	1	0	0
	4	1	1	1	x	1	1	1	1	1	0
Actor	5	1	0	1	1	x	0	0	1	0	1
	6	0	1	1	1	0	x	1	1	1	0
	7	1	1	1	1	0	1	x	1	0	0
	8	0	1	1	1	1	1	1	x	1	1
	9	1	1	0	1	0	1	0	1	x	0
	10	0	0	0	0	1	0	0	1	0	x

Asymmetric

Partner

		1	2	3	4	5	6	7	8	9	10
	1	x	1	1	0	1	0	1	0	1	0
	2	1	x	1	1	0	0	0	1	0	0
	3	1	1	x	1	1	1	1	1	0	0
	4	0	1	1	x	0	1	1	0	0	0
Actor	5	1	0	1	0	x	1	1	1	1	0
	6	0	1	1	1	1	x	1	1	1	0
	7	1	0	1	1	0	1	x	0	0	0
	8	0	1	1	1	1	0	0	x	1	0
	9	1	1	0	1	1	1	0	1	x	0
	10	0	1	1	1	1	1	1	1	1	x

Very Asymmetric

Partner

		1	2	3	4	5	6	7	8	9	10
	1	x									
	2	1	x	1	1	1	0	0	1	0	0
	3	1	0	x	1	1	1	1	1	1	0
	4	1	1	1	x	0	1	1	1	0	0
Actor	5	1	0	1	1	x	1	1	1	1	0
	6	0	1	1	1	0	x	1	1	0	0
	7	1	1	1	1	0	1	x	1	1	0
	8	0	0	0	0	1	0	0	x	0	0
	9	1	1	0	1	0	1	1	1	x	0
	10	1	1	1	1	1	1	1	1	1	x

Table 2

Nine SRM Population Parameter Configurations

	Configuration								
	1	2	3	4	5	6	7	8	9
Actor Var in X	2	2	0	0	2	2	2	2	2
Partner Var in X	1	1	0	0	1	1	1	1	1
Actor-Partner Cov in X	0	-.707	0	0	0	-.707	0	-.707	-.707
Relationship Var in X	0	0	7	7	7	7	7	7	7
Dyadic Cov in X	0	0	0	0	0	0	0	0	-1.75
Actor Var in Y	0	0	0	0	0	0	0	0	3
Partner Var in Y	5	5	0	0	5	5	5	5	5
Actor-Partner Cov in Y	0	0	0	0	0	0	0	0	1.936
Relationship Var in Y	0	0	20	20	20	20	20	20	20
Dyadic Cov in Y	0	0	0	10	0	0	10	10	10
Actor-Actor XY Cov	0	0	0	0	0	0	0	0	-.612
Partner-Partner XY Cov	0	0	0	0	0	0	0	0	-1.118
Actor-Partner XY Cov	0	1.581	0	0	0	1.581	0	1.581	1.581
Partner-Actor XY Cov	0	0	0	0	0	0	0	0	.577
Dyad[i,j]-Dyad[i,j] XY Cov	0	0	0	-8.282	0	0	-8.282	-8.282	-8.282
Dyad[i,j]-Dyad[j,i] XY Cov	0	0	0	0	0	0	0	0	1

Table 3

Deviation between Sample Estimates and Population Parameter Values

Parameter	Mean Deviation	Mean Percent Deviation	Mean Absolute Percent Deviation[1]
Actor Variance in X	.0003	+0.01%	0.28%
Partner Variance in X	-.0004	-0.04%	0.52%
Actor-Partner Covariance in X	-.0006	-0.54%	0.93%
Relationship Variance in X	.0011	+0.02%	0.10%
Dyadic Covariance in X	.0021	+0.18%	0.22%
Actor Variance in Y	-.0002	-0.26%	0.40%
Partner Variance in Y	.0065	+0.15%	0.34%
Actor-Partner Cov in Y	.0036	-0.99%	1.41%
Relationship Var in Y	-.0050	-0.03%	0.11%
Dyadic Covariance in Y	-.0072	-0.06%	0.34%
Actor X – Actor Y Cov	-.0011	-0.48%	0.95%
Partner X -Partner Y Cov	.0009	+0.16%	0.77%
Actor X - Partner Y Cov	-.0014	-0.05%	0.62%
Actor Y – Partner X Cov	.0003	+2.59%	2.59%
Dyad X(i,j) Dyad Y(i,j) Cov	.0022	+0.02%	0.10%
Dyad X(I,j) Dyad Y(j,i) Cov	-.0017	-1.04%	1.20%

[1] Percent deviations are for non-zero population parameter values only.

Table 4

Asymmetry and Estimation Error

	Within-effect Parameters	Actor-Partner Covariances	Dyadic Covariances
Symmetric Data	0.28%	0.66%	0.32%
	(1.85	1.90	2.11)
Asymmetric Data	0.23%	1.34%	0.83%
	(1.89	2.29	4.68)
Very Asymmetric Data	0.36%	1.79%	0.52%
	(2.11	3.57	6.93)

Note: The entries in the Table are mean absolute percentage estimation error (on the first row of each pair) and mean observed standard deviation in parameter estimates (in parentheses, in the second row of each pair).

Table 5

Mean Absolute Differences in Observed and Theoretical Standard Deviations Across Two Data Batches

Mean Absolute Difference between: and:	Observed A Observed B	Theoretical A Theoretical B	Observed A Theoretical B	Theoretical A Observed B
Dyadic Structure				
Round-robin	.0122	.0045	.0082	.0086
Concentrated symmetric	.0135	.0052	.0110	.0100
Diffuse symmetric	.0123	.0043	.0087	.0085
Asymmetric	.0138	.0052	.0090	.0126
Very asymmetric	.0343	.0150	.0247	.0242

Table 6

Observed and Theoretical Standard Deviations in SRM Parameter Estimates

Dyadic Structure	Mean Observed SD	Mean Theoretical SD	Mean Absolute Observed/Theoretical Difference	Mean Percentage Absolute Observed/Theoretical DIfference
Round Robin	1.6833	1.6840	.0038	0.20%
Concentrated Symmetric	1.8453	1.8451	.0053	0.30%
Diffuse Symmetric	1.8297	1.8297	.0040	0.25%
Asymmetric	2.3098	2.3111	.0065	0.28%
Very Asymmetric	3.6556	3.6546	.0080	0.25%